

PSYC402-week-18-LME-2

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Targets for Week 18 – Ideas and skills

- 1 Practice how to tidy experimental data for mixed-effects analysis
- 2 Begin to develop understanding of crossed random effects of subjects and stimuli
- 3 Practice fitting linear mixed-effects models incorporating random effects of subjects and stimuli

To be modern psychological data analysts you will need to know the *what, why, when and how* of multilevel or mixed-effects models

This week, we make a *subtle* change and start talking more about **Linear Mixed-effects models**

Repeated measures data: we begin by *revising* our list of when we need mixed-effects models

- When we test the same people multiple times
 - Pre- and post-treatment
 - Multiple stimuli – everyone sees the same stimuli
 - Repeated testing – follow learning, development within individuals – in longitudinal designs
- When we do multi-stage sampling
 - Find (sample) classes or schools – test (sample) children within classes or schools
 - Find (sample) clinics – test (sample) patients within clinics

Where we are going: linear mixed-effects models

- We need to learn how to estimate the effects of experimental variables
- *while also* taking into account sources of error variance like
 - the random differences between people we test
 - and the random differences between stimuli we present

The wider scientific impact – accepting diversity

- How do psychological effects *vary*?
- Uniformity is a common because convenient assumption
- We ask: *How do people vary in their response?*



The data we will work with: the CP study data

- As part of our lab work, we will practice steps often required to get data ready for mixed-effects model
- CP studied how 62 children read 160 words
- The data are in separate files and the files are *untidy*
 - CP study word naming rt 180211.dat reaction time for correct responses to word stimuli in reading
 - CP study word naming acc 180211.dat accuracy for all responses to word stimuli in reading
 - words.items.5 120714 150916.csv information about the 160 stimulus words presented in reading task
 - all.subjects 110614-050316-290518.csv information about the 62 participants

We will make data tidy

- What a horrible mess:
 - Psychological data collection often delivers *untidy* data
 - Here, we have data for different participants in separate columns
 - Each row holds the reaction times for the responses made by all participants to each stimulus word
 - Each cell holds the reaction time for the response made by a child to a word
 - We have missing values *NA* and reaction times

```
## # A tibble: 6 x 62
##   item_name AislingoC AlexB AllanaD AmyR AndyD AnnaF AoifeC
##   <chr>      <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 act        595.  586    NA   693  597  627  645
## 2 ask        482.  864  1163  694.  616  631  535
## 3 both       458.  670  1114.  980  1019  796.  545
## 4 box        546  749.   975  678  589  604  574
## 5 broad      580 1474.   NA   789  684  NA   815
```

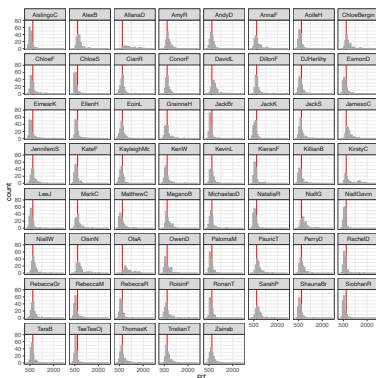
Next: When do we need mixed-effects models?

When do we need mixed-effects models? *When we have repeated measures data*

- In a reading study, we ask all individuals in a participant sample to read all words in a stimulus sample
- For each individual, we will have multiple observations and these observations will not be independent
 - One participant will tend to be slower or less accurate compared to another
 - Her responses may be more or less susceptible to the effects of the experimental variables
- The observed responses in different trials can be grouped by participants

Participants will vary for reasons we cannot explain

- Here you see a separate histogram plot for each participant
- Bars show the distribution of reaction time (RT)
- The red line shows the overall mean RT

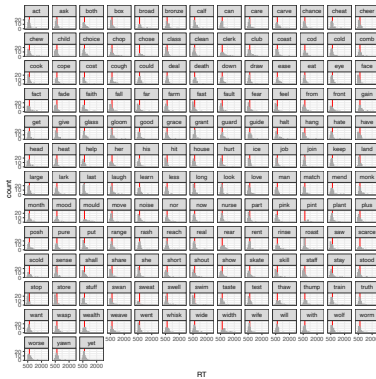


When do we need mixed-effects models? *When we have repeated measures data*

- In a reading study, we ask all individuals in a participant sample to read all words in a stimulus sample
- For each stimulus, there are multiple observations and these observations will not be independent
 - One stimulus may prove to be more challenging to all participants compared to another, eliciting slower or less accurate responses
 - The effects of *within-items* experimental variables may be more or less prominent for responses to some stimuli than to others
- So the data can *also* be grouped by stimuli

Stimuli will vary for reasons we cannot explain

- Here you see a separate histogram plot for the responses to each word
- Bars show the distribution of reaction time (RT)
- The red line shows the overall mean RT



The language-as-fixed-effect fallacy

- If you are doing a **repeated measures** design study in which there are different participants
- And different tests or test items or stimuli
- And all participants respond to all stimuli
- **Then you need to use mixed-effects models**
- Because you need to deal with the random differences between people *and* the random differences between stimuli

The language as fixed effect fallacy

A very famous paper by Clark (1973)

- Historically, psychologists tested effects against error variance due to differences between people
- They ignored differences due to stimuli
- This meant they were likely to find significant effects not because there were true differences between conditions
- But because there were random differences between stimuli presented in different conditions

Taking into account error variance due to subjects and items – Clark's (1973) $minF'$ solution

$$minF' = \frac{MS_{effect}}{MS_{random-subject-effects} + MS_{random-word-differences}} = \frac{F_1 F_2}{F_1 + F_2} \quad (1)$$

- 1 You start by *aggregating* your data
 - By-subjects data – for each subject, take the average of their responses to all the items
 - By-items data – for each item, take the average of all subjects' responses to that item
- 2 You do separate ANOVAs, one for by-subjects (F1) data and one for by-items (F2) data
- 3 You put F1 and F2 together, calculating $minF'$

Using tidyverse functions, it is easy to calculate by-subjects and by-items RT averages

```
by.items.rt <- long.all.noNAs %>%
  group_by(item_name) %>%
  summarise(av_RT = mean(RT, na.rm = TRUE))

by.items.rt

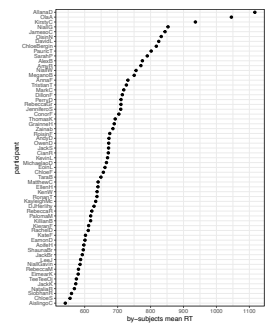
by.subjects.rt <- long.all.noNAs %>%
  group_by(subjectID) %>%
  summarise(av_RT = mean(RT, na.rm = TRUE))

by.subjects.rt
```

- We can then join the by-items data with stimulus properties and analyze the effects of those properties (e.g. word frequency)
- or we can join the by-subjects data with participant attributes and analyze the effects of those attributes (e.g. participant group)
- We cannot look at *both* item and participant effects at the same

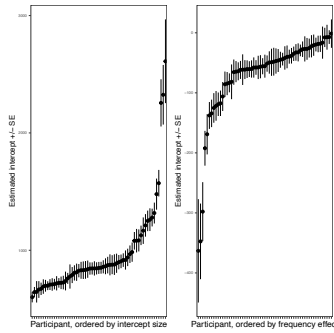
But analysing data only by-items means we lose track of participant differences

- *Lorch & Myers (1990)* warn: analyzing just by-items mean RTs assumes wrongly that *subjects are a fixed effect*
- We can see this is wrong because, for example, with the CP data, we can see that participant RT varies substantially



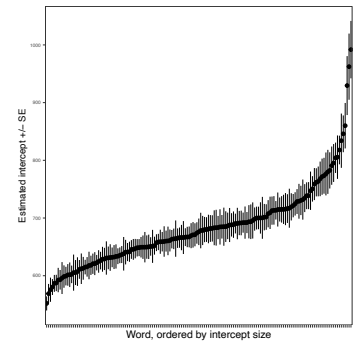
Participant differences in *both* average RT (or accuracy) *and* the impacts of effects

- These error bar plots show:
 - As points: the estimated intercept or the estimated effect of frequency on RT
 - Together with the standard errors of the estimates
 - For each participant analyzed separately
- We can see that participants vary greatly in both estimated intercept or slope *and* in uncertainty about estimates



Equally, analysing by-subjects data alone means we would lose track of random differences between stimuli

- These error bar plots show:
 - As points: the estimated intercept
 - Together with the standard errors of the estimate
 - For responses to each word analyzed separately
- We can see that responses to different words vary greatly in average speed – here, we ignore other effects



Next: So what do we do? We use mixed-effects models and we include random effects for both participants and stimuli

We account for differences between participants in intercept by modelling the intercept as two terms

$$\beta_{0i} = \gamma_0 + U_{0i} \quad (2)$$

- Where γ_0 is the average intercept
- And U_{0i} is the difference for each i child between *their intercept* and the average intercept

We account for differences between participants in slope by modelling the slope of effects as two terms

We account differences between items in intercepts by modelling the intercept as two terms

$$\beta_{1i} = \gamma_1 + U_{1i} \quad (3)$$

- Where γ_1 is the average slope
- And U_{1i} represents the difference for each i child between the slope of *their frequency effect* and the average slope

$$\beta_{0j} = \gamma_0 + W_{0j} \quad (4)$$

- Where γ_0 is the average intercept
- And W_{0j} represents the deviation, for each word, between the *word intercept* and the average intercept

Our model can now incorporate the random effects of *both* participants and words

$$Y_{ij} = \gamma_0 + \gamma_1 X_j + U_{0i} + U_{1i} X_j + W_{0j} + e_{ij} \quad (5)$$

- Where the outcome Y_{ij} is related to ...
- The average intercept γ_0 and differences between i children in the intercept U_{0i} ;
- The average effect of the explanatory variable frequency $\gamma_1 X_j$ and differences between i participants in the slope $U_{1i} X_j$;
- Plus the random differences between items in intercepts W_{0j}
- And the residual error variance e_{ij} .

We can do all this in one move using `lmer()`

```
lmer.all <- lmer(RT ~ Lg.UK.CDcount +
  (Lg.UK.CDcount + 1||subjectID) +
  (1|item_name),
  data = long.all.noNAs)
summary(lmer.all)
```

We can do all this in one move using `lmer()`

```
lmer.all <- lmer(RT ~ Lg.UK.CDcount +
  (Lg.UK.CDcount + 1||subjectID) +
  (1|item_name),
  data = long.all.noNAs)
summary(lmer.all)
```

- `lmer.all <- lmer(...)` create a linear mixed-effects model object using the `lmer()` function
- `RT ~ Lg.UK.CDcount` the fixed effect in the model is expressed as a formula in which the outcome RT is predicted ~ by word frequency, given by `Lg.UK.CDcount` in the dataset
- We use `data = long.all.noNAs` to specify the data we are analyzing

We can do all this in one move using `lmer()`

```
lmer.all <- lmer(RT ~ Lg.UK.CDcount +
  (Lg.UK.CDcount + 1||subjectID) +
  (1|item_name),
  data = long.all.noNAs)
summary(lmer.all)
```

- We add `(...|subjectID)` to specify random differences between sample groups (here, participants), specified using the dataset `subjectID` coding variable name
- We add `(...1|subjectID)` to account for random differences between participants in intercepts, coded `1`
- and `(Lg.UK.CDcount ... |subjectID)` to account for random differences between participants in the slope of the frequency effect, specified using the dataset `Lg.UK.CDcount` variable name

We can do all this in one move using `lmer()`

```
lmer.all <- lmer(RT ~ Lg.UK.CDcount +
  (Lg.UK.CDcount + 1||subjectID) +
  (1|item_name),
  data = long.all.noNAs)
summary(lmer.all)
```

- We add the term `(...|itemname)` to specify random effects corresponding to random differences between sample groups (here, items) coded using the `itemname` variable name
- We add `(1|itemname)` to account for random differences between sample groups (words) in intercepts, coded `1`

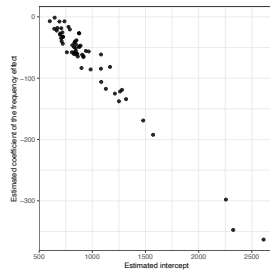
We usually do not aim to examine the specific deviations

We estimate just the *spread of deviations* by-participants or by-words: the **variance**

- $var(U_{0i})$ variance of deviations by-participants from the average intercept;
- $var(U_{1i} X_j)$ variance of deviations by-participants from the average slope of the frequency effect;
- $var(W_{0j})$ variance of deviations by-items from the average intercept;
- $var(e_{ij})$ residuals, at the response level, after taking into account all other terms.

Expect random effects will covary

- Participants who are slower to respond also show the frequency effect more strongly
- The scatterplot shows the relationship between per-participant estimates of
- The intercept and the slope
- The strong relationship is clear



How do you report a mixed-effects model?

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: RT ~ Lg.UK.CDcount + (Lg.UK.CDcount + 1 || subjectID) + (1 |
## item_name)
## Data: long.all.noMAs
##
## REML criterion at convergence: 116976.7
##
## Scaled residuals:
##   Min       1Q   Median       3Q      Max
## -4.1794 -0.5474 -0.1646  0.3058 12.9485
##
## Random effects:
##   Groups Name Variance Std.Dev.
##   item_name (Intercept) 3397  58.29
##   subjectID Lg.UK.CDcount 3623  60.20
##   subjectID.1 (Intercept) 112307  335.12
##   Residual 20704  143.89
## Number of obs: 9085, groups: item_name, 159; subjectID, 61
##
## Fixed effects:
##              Estimate Std. Error   df t value Pr(>|t|)
## (Intercept)  971.07      51.86  94.62  18.723 < 2e-16 ***
## Lg.UK.CDcount -72.33      10.79  125.27  -6.703 6.29e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr)
## Lg.UK.CDcnt -0.388
```

How do you report a mixed-effects model?

- Explain what variables went into the analysis: say what the outcome and predictor variables were
- Report the model equation $RT \sim \text{frequency} + (\text{frequency} + 1 || \text{participant}) + (1 | \text{word})$
- Report a table of coefficients: variable, estimate of coefficient of effect; SE; t (or z); and p
- Add to that table a report of the random effects terms: variances
- You should comment on the coefficient estimates; you may (or may not) comment on the random effects variances

Next week: we need to be ready to trouble shoot

- I stopped the model from estimating the covariance between random effects of participants on items and on slopes
- using $(\text{frequency} + 1 || \text{participant})$ not $(\text{frequency} + 1 | \text{participant})$
- next week I explain why: convergence

```
library(lmerTest)
lmer.all <- lmer(RT ~ Lg.UK.CDcount +
  (Lg.UK.CDcount + 1 || subjectID) +
  (1 | item_name),
  data = long.all.noMAs)
summary(lmer.all)
```

Summary – Week 18: crossed random effects

- Psychological studies often have repeated measures designs
 - When there are multiple observations for each person or stimulus
 - Because each person has to respond to multiple stimuli
 - And each stimulus is shown to multiple people
- Mixed-effects models can be specified by the researcher
 - to account for random differences between participants or between stimuli
 - in the intercepts or the slopes of explanatory variables

Human diversity and how people vary: the challenge, the promise

- Variation is a fact and mixed-effects models enable us to take into account random differences between people
- But these models also allow us – this is new – to examine the nature of the variation directly

